Psychometric workup of instruments: Early phase and EFA

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1st December, 2558

What we will cover....

1. Introduction
2. Early phase: Developing content
   - Definition and approaches
   - Case study of Delphi technique
3. Middle phase: Exploring instrument structure
   - Purpose of EFA
   - Statistical methods for EFA

The work up of psychometric/clinimetric instruments: the validation process

- Generally involves the use of survey data to (ultimately) measure psycho-social constructs using (sub)scales
- Items (questions) in the survey need to be on an ordinal scale
  - strongly disagree, disagree, ..., strongly agree
  - never, sometimes, ..., always
- Items are combined to form scales (representing constructs) which can then be used as (1) health predictors or outcomes to answer important research questions, or (2) Diagnostic tools in their own right
- Why bother? Because without this workup these psycho-social constructs would be otherwise unmeasurable, or otherwise poorly measured

Terminology

- Lots of words bandied about (inconsistent usage)
  - Concurrent validity
  - Construct validity
  - Content validity
  - Convergent validity
  - Criterion-based validity
  - Discriminant validity
  - Divergent validity
  - Face validity
  - Predictive validity
  - Translational validity
- I have found a lot of disagreement in texts/literature. I will use definitions that seems to have the most consensus

Terminology

Personally I have found it useful to categorize into three categories:

1. Early phase validation: Devising the items
   - Content validity
   - Face validity
   - Translational validity
2. Middle phase validation: From items to (sub)scales
   - Construct validation
3. Final phase validation: Gauging new scales against existing(gold standard) instruments
   - Criterion-based validity

Early phase: Devising the items

- Def^n: Items = survey questions
- This phase is generally **qualitative** in nature
- Generally involves building and refining pools of items based on **literature**, **focus groups** and/or **panels of experts**. Several approaches:
  1. Initial focus groups to generate large pool of questions that are successively refined using expert panels
  2. **Delphi method**: Iterative series of expert panels to define and refine items to be included
  3. Some other combination of the above approaches

Early phase: Content and face validity

- Once our panel of experts is happy with the items, the instrument is said to be **CONTENT VALID**
- Once the panel is happy with flow (order) of items, grammar etc, the instrument is said to have **FACE VALIDITY**

Often an existing instrument (e.g. used on another population) can be translated (e.g. English → Thai). The instrument is forward- and then back- translated. If done satisfactorily, the instrument is said to have **TRANSLATIONAL VALIDITY**. This is really just a special case of face validity.

Case study: Vaccination of children

- In Australia, childhood vaccination is not mandatory, even though the federal government offers financial incentives to parents to get their children vaccinated
- In recent years there has a decline in parents getting their children vaccinated, and we are interested in determining what factors might explain this decline
- A literature review reveals while studies have teased out motivators for, and barriers against, parents getting their children vaccinated, many of these studies were either unsystematic or uninformative about how they developed their methods for measuring parental attitudes and behaviour regarding child vaccination
- Even where the studies gave information about the instrument design, the instruments were only content validated.

Case study: Vaccination in children

- After reviewing 35 of the 'best' (international) studies that considered this topic, five domains were identified:
    1. Behaviour
    2. Knowledge
    3. Attitudes
    4. Beliefs
    5. Doubts and concerns
- After consulting with two experts (Health Psychologist and Social Epidemiologist), we decided that **Behaviour**, **Knowledge** and **Beliefs** are sufficient to measure the motivators and barriers to childhood vaccination

Case study: Vaccination in children

Delphi technique

- We decide to conduct a 3 to 5 round Delphi study
- 30 experts (including health psychologists, family doctors, paediatricians and social epidemiologists) are invited to participate in the expert panel, 20 of which agree

ROUND ONE

- We send out a list of 80 items (from the literature) and ask the experts to consider the suitability of the individual items and to classify them into one of the three domain (Behaviour, Knowledge, Beliefs)
- Of the 20 experts consulted, only 18 make the deadline
- We decide to only include items where there is at least 60 percent consensus on their suitability (for inclusion) and domain membership
- Only 30 of the original items meet the above criteria

Case study: Vaccination in children
Delphi technique

ROUND TWO

- We send out the list of 30 items classified into their three domains (Behaviour, Knowledge and Beliefs) seeking agreement regarding which items are included, and whether experts agree with the item domain memberships
- From the 17 remaining experts, a large majority of items demonstrate at least 80 percent consensus
- Some experts comment on the redundancy of some items, but we decide to keep them as a measure of reliability
- Some experts also suggest the inclusion of a few more items in the behaviour domain
- After adding in two items (suggested by experts) and excluding 4 others (with low consensus), we have a 28 item instrument

Case study: Vaccination in children
Delphi technique

ROUND THREE

- In the third round, we ask the experts to pay particular attention to both the WORDING and SEQUENCING of the questions.
- A few experts point out that the language used in some of the items is a little technical and suggest amendments that simplify the meaning of the items (for our target population).
- One expert (not well versed in instrument development and it's purpose) suggests changing some of the questions to open answer questions. We ignore this advice.

Case study: Vaccination in children

Delphi technique

ROUND FOUR

- After incorporating much of the advice from round three we send out the proposed survey instrument (including instructions to participants) to the 17 experts
- We also provide demographic questions that will also be included (Socio-economic, Age, Rurality etc). These will not be included in the measurement instrument itself, but will represent covariates that may explain **Behaviour**, **Knowledge** and **Beliefs** in a planned study.
- We find that there is strong consensus and satisfaction among the experts and we make some suggested amendments to wording of the demographic questions
- We conclude that we now have an instrument that is both **CONTENT** and **FACE** valid

Middle phase: From items to sub-scales

- We will now enter into the more quantitative side of measurement instrument development and validation
- I will cover two (mathematically) closely related approaches:
  1. Exploratory Factor Analysis (EFA) - this session; and
  2. Confirmatory Factor Analysis (CFA) - next session
- Explanatory factor analysis involves a couple of iterative steps with a view to develop a instrument that can be subsequently (construct) validated using CFA
- EFA is about deriving some scales (based on groupings of associated items) that will hopefully 'theme' with some 'nameable' construct (e.g. Behavior, Knowledge etc)
- IN A NUTSHELL, this step should identify the **number** and **nature** of the sub-scales (or **'Factors'**) that are hopefully measured by our instrument

Best practice Vs. Practicality

- Ideally, a separate pilot sample should be used for EFA, and then a larger sample should be used for the CFA
- However, in many situations we are just presented with a sample to use for both steps
- If we have only a single sample, data-splitting should be used
- Factor analysis is a little to complicated for a meaningful sample size calculation to be performed. EFA is also not inferential so significance and power are meaningless.
- I have seen a couple of guidelines presented:
    1. n = 60 + 5k
    2. n = (5 to 20)k

where k is the number of items

My experience has been that with a typical 25+ item instrument, EFA should not have $N < 150$.

Factor Analysis

- Factor analysis (EFA and CFA) are a type of **multivariate** analysis (i.e. multiple dependant variables)
- Multivariate techniques take advantage of the redundancy inherent in multiple correlated variables to simplify their structure
- For our purpose here, we are trying to come up with a smaller number of dimensions (e.g. Behaviour, Knowledge, Beliefs) based on our much larger number of items
- In this sense we are 'dimension reducing' the data

Building blocks: A typical dataset used for EFA

Participant	Q_1	Q_2	Q_3	$\cdots$	$\cdots$	Q_{k-1}	Q_k
1	4	3	4	$\cdots$	$\cdots$	1	3
2	3	3	5	$\cdots$	$\cdots$	4	2
3	5	4	4	$\cdots$	$\cdots$	2	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$n-1$	3	2	3	$\cdots$	$\cdots$	3	5
n	3	3	4	$\cdots$	$\cdots$	5	4

Where: *1=Strongly disagree; 2=Disagree; 3=Neutral; 4=Agree; 5=Strongly agree*

Question: Can items be separated into homogeneous groups (of questions) that collectively represent some broader (and currently unobserved) construct?

Building blocks: The Covariance and Correlation matrices

- Given a large number of items, we can consider all pair-wise correlations (or covariances) among items in a matrix.
- It is this **association matrix** that represents the key unit of analysis in Factor Analysis.

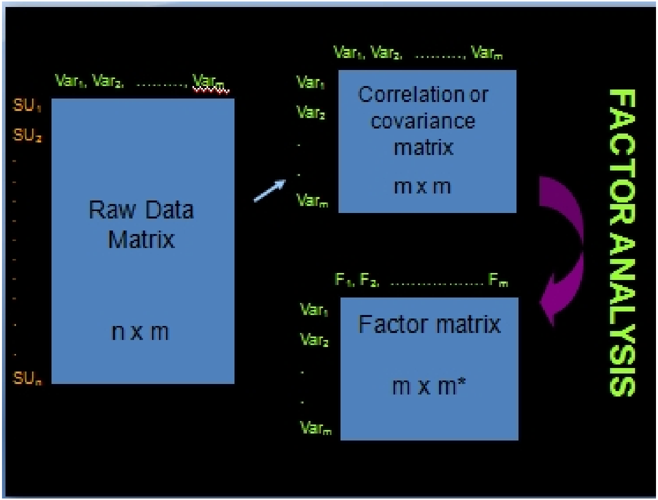
Example of a correlation matrix Correlation matrix containing three PHYSICAL and three SOCIAL well being items Ideally 'Within-domain' item correlations should be higher than 'Between-domain' item correlations

	PWB1	PWB2	PWB3	SWB1	SWB2	SWB3
PWB1	1.0000	0.3099	0.3487	0.1609	0.1511	0.1608
PWB2	0.3099	1.0000	0.3975	0.0469	0.0907	0.0512
PWB3	0.3487	0.3975	1.0000	0.1070	0.1009	0.1184
SWB1	0.1609	0.0469	0.1070	1.0000	0.5435	0.7707
SWB2	0.1511	0.0907	0.1009	0.5435	1.0000	0.6722
SWB3	0.1608	0.0512	0.1184	0.7707	0.6722	1.0000

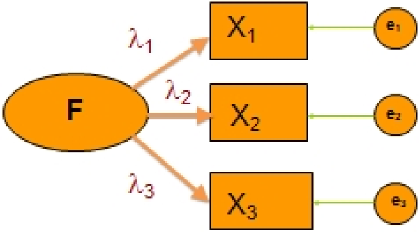
R code:

```
#A subset of 6 variables from the larger set
small.df<-factanal.df[,c(1:3, 7:9)]
#Generate correlation matrix
cor(small.df)
```

Building blocks of factor analysis



Schematic of a One Factor model



Note the **direction** of the arrows

In Schematic...

- **F** represents a (hitherto) latent variable
- X_i represents the i^{th} measured variable (e.g. Item on the questionnaire)
- λ_j are Factor loadings of each of the measured variables
- e_i is the residual associated with X_i (that not explained by the factor)

In one factor model: **Loading = Corr(Factor, Item)**

Again, **NOTICE** the direction of the arrows. F is the explanatory variable and the Xs are the outcome (response) variables.

Formulation of the One Factor Model

Measured variables should be allowed to have varying 'associations' with the factor

$$X_1 - \mu_1 = \lambda_1 F + e_1$$
$$X_2 - \mu_2 = \lambda_2 F + e_2$$
$$\vdots$$
$$X_k - \mu_k = \lambda_k F + e_k$$

In other words there is unequal 'sensitivity' among Xs to change the factor.
Note that subtracting each variable's mean ($\mu.$) just centres that variable.

Extending the model to > 1 factors

- In most situations, there are multiple factors (constructs) underlying a set of variables
- If we assume the factors are not correlated (they are orthogonal)...

For m factors and k measured variables ($m \leq k$):

$$X_1 - \mu_1 = \lambda_{1,1} F_1 + \lambda_{1,2} F_2 + \cdots + \lambda_{1,m} F_m + e_1$$
$$X_2 - \mu_2 = \lambda_{2,1} F_1 + \lambda_{2,2} F_2 + \cdots + \lambda_{2,m} F_m + e_2$$
$$\vdots$$
$$X_k - \mu_k = \lambda_{k,1} F_1 + \lambda_{k,2} F_2 + \cdots + \lambda_{k,m} F_m + e_k$$

A matrix representation

$$\begin{bmatrix} x_{.1} - \mu_1 \\ x_{.2} - \mu_.2 \\ \vdots \\ x_{.k} - \mu_k \end{bmatrix} = \begin{bmatrix} \lambda_{1,1} & \lambda_{2,1} & \cdots & \lambda_{m,1} \\ \lambda_{1,2} & \lambda_{2,2} & \cdots & \lambda_{m,2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{1,k} & \lambda_{2,k} & \cdots & \lambda_{m,k} \end{bmatrix} \begin{bmatrix} F_{.1} \\ F_{.2} \\ \vdots \\ F_{.m} \end{bmatrix} + \begin{bmatrix} \epsilon_{.1} \\ \epsilon_{.2} \\ \vdots \\ \epsilon_{.k} \end{bmatrix}$$

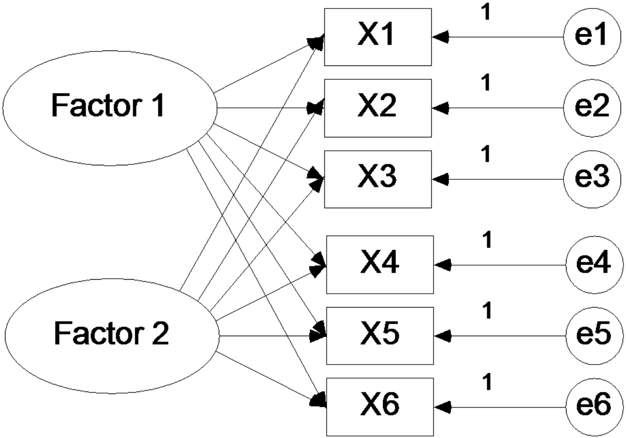
or, more concisely,

$$X' = LF + \epsilon$$

Where X' are the (mean) centred observations of the X variables.

We call L the factor loading matrix (and as above it represents the association between the individual variables and factors)

Schematic of a two factor model



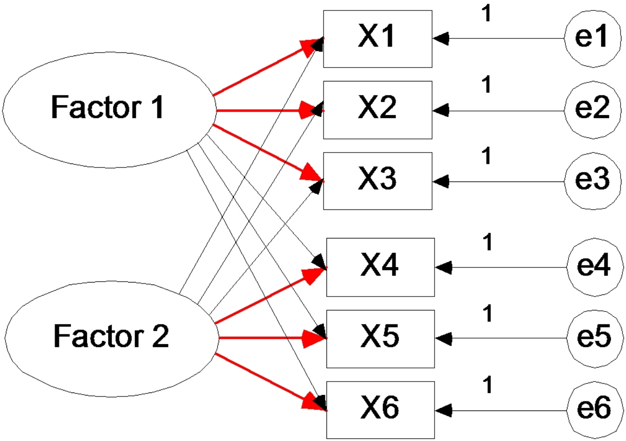
It should be noted that in EFA (in contrast to CFA), **every item** potentially loads on **every factor**

Noteworthy properties of Factors in EFA

- Each Factor assumes a measured variable (X) has the potential of some (non zero) loading
    - That is, each variable (item) loads of each factor
    - Loadings represent factor-item correlation: $corr(F_j, X_{ij})$
- In Exploratory Factor Analyses we want to see patterns in how items load
    - That is, we groups items that are associated with each factor
- AND we want items to group in a meaningful (and interpretable) way
- THAT IS: a way that aligns with our (theoretical) notions -> constructs (domains)

A 'successful' Exploratory Factor analysis

Here the **RED** arrows represent 'substantial' loadings
e.g. $|\lambda| > 0.4$



Final word (for now) on the maths underlying the Factor-analytic model

- Eigenanalysis is used to estimate the loadings (i.e. $\lambda_{i,j}s$)
- Eigenanalysis is a set of matrix operation that allows us to fit the factor analytic model on the last slide.
- Idea:
  1. Each X can be represented (i.e. predicted) by a linear combination of the factors (see model a few of slides back)
  2. Produces new variable (components or factors) that are mutually linearly independent (uncorrelated), and are based on our original items

Steps in (Vanilla) Exploratory Factor Analysis

1. Collect and explore data: choose relevant items (early phase work-up).
2. **Extract initial factors (via Principal Components Analysis)**
3. **Choose number of factors to retain (Kaiser criterion, Scree plot, Parallel analysis)**
4. **Choose Extraction method (e.g. Principal Axis Factoring)**
5. **Rotate and interpret (Orthogonal? Oblique?)**
6. **Assess whether model 'acceptable', if not decide if changes can be made at the:**
    - **'input' level (variables included or excluded) - step 1**
    - **'technique' level (estimation and/or rotation) - steps 4 and 5**
7. 'Name' scales and go on tonext step (i.e. CFA)

Choices in Factor Analysis

Once items have been selected there are two main choices to make:

Step 4: Extraction method - 'the method / model'

Step 5: Rotation - improving the interpretablity

Extraction method

- A better way to think of the extraction method is the 'method' of EFA
- Mathematically simplest (and original type) is Principal Components Analysis (PCA) - not a very good method for factor analysis (but useful as a starting point)
- Principal Axis Factoring (PAF) aka Common Factor Analysis is the most widely used method and will suit you in most situations
- Others variants available based more on various statistical estimation algorithms (e.g. Maximum Likelihood Estimation, Generalized Least Squares, etc.)

Rotation

- Given a solution (initial factors), we can rotate that solution to aid in interpretation (i.e. to reveal or fit factors)
- Can think of this step as aligning our construct (science) with the main axes (directions) of variation in the data
- Two main families:
  1. Orthogonal (e.g. Varimax, Quartimax and Equimax)
  2. Oblique (e.g. Oblimin and Promax)

Motivating example: A naive EFA

- Let's get our hands dirty with some data.
- Fictitious dataset including 20 of the Functional Assessment of Cancer Therapy-General (FACT-G) items including:

- Six Physical QoL items
- Four Emotional QoL items
- Four Social QoL items
- Six Functional QoL items

Examples of some of the items from each 'domain'

- **Physical well being**
  - PWB1 I have a lack of energy
  - PWB5 I am bothered by the side effects of my treatment
- **Emotional well being**
  - EWB1 I feel sad
  - EWB4 I feel nervous
- **Social well being**
  - SWB3 I feel close to my friends
  - SWB4 My family has accepted my illness
- **Functional well being**
  - FWB4 I am able to enjoy life
  - FWB5 I am sleeping well

All of these items are scored:
Never, Rarely, Sometimes, Often, Always

Determining the number of factors

- To help understand the process of exploratory factor analysis, let's use this dataset
- Before we start, let's consider the questions
  - 'Theoretically', how many factors should be revealed in this data??
  - Would we expected these factors to be correlated??

Empirically, we can determine the **NUMBER** of factors using Principal Components Analysis (PCA)

Principal Components Analysis in R

```
#Library used for parallel analysis (see later)
library(nFactors)

#Read in data
setwd("D:\\Clinometric\\EarlyAndEFA\\Data")
all.df<-read.csv("RealLifeCFAdataset.csv" )

#Trim out excess variables:ID, Gender, Location
factanal.df<-all.df[,-c(1, 2 ,23,24)]


#Use PCA to determine number of factors
EFA.PCA<-princomp(factanal.df, cor = TRUE)
summary(EFA.PCA)
```

```
Importance of components:
                       Comp.1 Comp.2 Comp.3 Comp.4 Comp.
Standard deviation     2.5579 1.6003 1.1952 1.1081 0.933
Proportion of Variance 0.3271 0.1280 0.0714 0.0614 0.043
Cumulative Proportion  0.3271 0.4552 0.5266 0.5880 0.631
                       Comp.6 Comp.7 Comp.8 Comp.9 Comp.
Standard deviation     0.9243 0.8567 0.8429 0.8186 0.774
Proportion of Variance 0.0427 0.0367 0.0355 0.0335 0.030
Cumulative Proportion  0.6743 0.7110 0.7465 0.7801 0.810
                       Cmp.11 Cmp.12 Cmp.13 Cmp.14 Cmp.1
Standard deviation     0.7536 0.7278 0.6737 0.6466 0.643
Proportion of Variance 0.0284 0.0264 0.0226 0.0209 0.020
Cumulative Proportion  0.8385 0.8649 0.8876 0.9085 0.929
                       Cmp.16 Cmp.17 Cmp.18 Cmp.19 Cmp.2
Standard deviation     0.6003 0.5666 0.5376 0.4999 0.439
Proportion of Variance 0.0180 0.0160 0.0144 0.0124 0.009
Cumulative Proportion  0.9473 0.9634 0.9778 0.9903 1.000
```

Even though PCA provides (simplistic) factors, we are more interested in determining the amount explained by each of the factors (table above).

A quick interpretation

- We can see the first component explains 32.7% of the total variation in the items
- The first component also an eigenvalue of 2.5579 (labelled standard deviation). We need to note each component's eigenvalue for later
- The second component has an eigenvalue of 1.6 and explains 12.8% of the variation
- Collectively the first two components explain 45.52%, and so on

Even though PCA is rarely used for true factor analysis, let's have a look at it's 'factor' anyway.

The first nine (of 20) components ('factors') from a principal components (Loading matrix)

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9
PWB1	-0.216	-0.145	-0.109		-0.201	-0.320		0.616	0.220
PWB2	-0.162	-0.296		0.275	0.189	-0.409	-0.469		
PWB3	-0.212	-0.304	0.202	0.208		0.109		0.214	
PWB4	-0.193	-0.222	0.223		-0.396	0.229	0.248	-0.255	-0.310
PWB5	-0.212	-0.340		0.254		-0.209	-0.114	-0.190	-0.198
PWB6	-0.169	-0.294	0.334		0.206		0.148		0.289
SWB1	-0.208	0.342		0.333	-0.182				0.336
SWB2	-0.204	0.322		0.370					-0.153
SWB3	-0.220	0.353		0.386	-0.119				0.198
SWB4	-0.177	0.276			0.469	-0.142	0.375	0.303	-0.377
EWB1	-0.231		-0.422	-0.139	0.180	-0.106	0.313	-0.290	
EWB2	-0.208	-0.110	-0.462		0.106		0.166	-0.320	0.328
EWB3	-0.141	-0.114	-0.446			0.512	-0.348	0.327	
EWB4	-0.228	-0.185	-0.221		-0.108	0.339	0.193		-0.279
FWB1	-0.235		0.281	-0.139	0.333	0.309		0.103	0.318
FWB2	-0.229	0.101	0.159	-0.337	0.217	0.152	-0.438	-0.207	
FWB3	-0.302	0.148		-0.212			-0.179		-0.185
FWB4	-0.235		0.115	-0.297	-0.443	-0.215			
FWB5	-0.300	0.118		-0.219	-0.134				
FWB6	-0.307	0.109		-0.216		-0.159			-0.252

A quick interpretation

- Again, we should note that PCA is not really useful as a factor analytic model itself (it is purely descriptive)
- But if we have a look at the factor loading matrix from the last slide we get an idea of how we might interpret the loadings in factor analysis
- We see for the first component that all items are weakly negatively correlated with the items (at best we could interpret this as weak scale for 'overall' quality of life (or rather lack of QoL)
- The second component is more interesting. We see physical and emotional well being are negatively associated, while social and functional WB are positive

Let's get back to the real reason we conducted a PCA. To determine the number of factors

Choosing the best number of factors

Number of methods:

1. Substantive theory -ideal
   - Here we have four 'suspected' factors: Physical, Social, Emotional and Functional well being
2. Kaiser criterion (Eigenvalue $>$ 1 rule)
   - Eigenvalue $=$ standard deviation(Factor)
   - Logic: Eigenvalue $>$ 1 implies factor explains more than any individual measured variable $\rightarrow$ improvement.
3. Scree plot: Plot showing us contribution of the factors
4. Parallel analysis
   - Method that tests null hypothesis: Eigenvalue ≤ 1
   - That is, eigenvalues need to be *significantly* > 1
   - Many journals (especially psychology literature) have now made this the preferred method for estimate correct number of factors

Example: Determining the number of factors

Method 1-Substantive theory:
We have agreed that we should see four factors emerging from the analysis (Physical, social, emotional, function)

Method 2-Kaiser Criterion:
Looking back at our PCA results (Importance of components), we see that the first four components have eigenvalues greater than 1 (suggesting a four factor model)

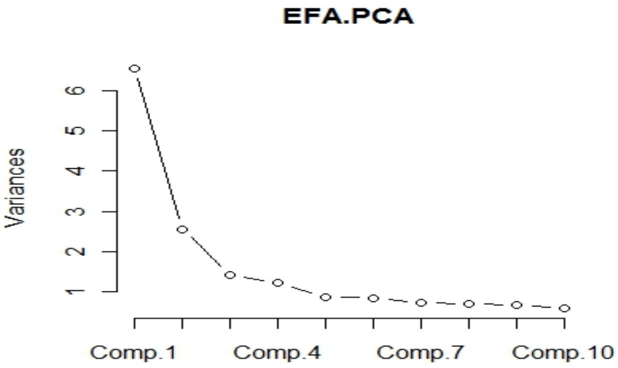
Method 3-Scree plots:
These are a simple plot where we are looking for a 'diminishing return' for adding additional factors

R code for scree plot

```
screeplot(EFA.PCA, type="lines")
```

Scree plot FACT-G data

Figure: Scree plot showing (absolute) amount of variation explained by each component



Parallel Analysis

The first three approaches for determining the best number of factors point towards a four-factor model. Let's see if Parallel analysis provides agreement.

R code for parallel analysis

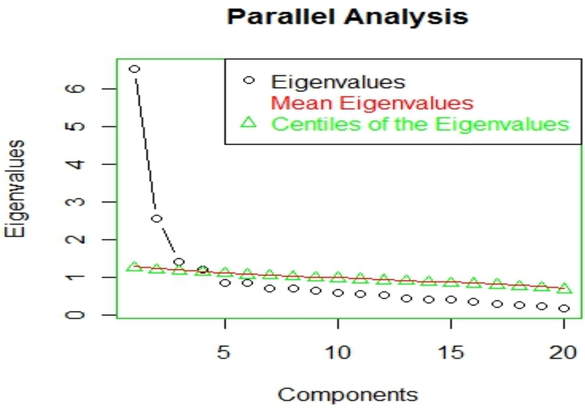
```
# get eigenvalues
ev <- eigen(cor(factanal.df))

# perform parallel analysis
ap <- parallel(subject = nrow(factanal.df),
+ var = ncol(factanal.df), rep=100, cent=.05)

#plot results on scree plot
plotParallel(ap, x=ev$values)
```

Parallel Analysis

Figure: Plot showing (boundary) of eigenvalues significantly > 1



Problems with PCA

PCA good for giving us idea about **number** of likely factors, but not good at the **factor structure** part. That is, the **nature** of the factors.

WHY??? PCA is really just a descriptive method with model:

$$X_1 - \mu_1 = \lambda_{1,1}F_1 + \lambda_{1,2}F_2 + \cdots + \lambda_{1k}F_k$$
$$X_2 - \mu_2 = \lambda_{2,1}F_1 + \lambda_{2,2}F_2 + \cdots + \lambda_{2,k}F_k$$
$$\vdots$$
$$X_k - \mu_k = \lambda_{k,1}F_1 + \lambda_{k,2}F_2 + \cdots + \lambda_{k,k}F_k$$

We should note:

- There are as many components as variables (items)
- There is no error term (variation in an item is assumed to be explained fully by the factor structure)
- The first few components that we deem 'important' (e.g. using Eigenvalue > 1 rule), we call **Principal** components

Principal Axis Factoring: A better factor analytic model

- Factor analysis is about explaining the shared variation among variables (the co-variation)
- However, the unique part of a variable contributes nothing when we are trying to parcel variables (based on variation they share)
- This collection of highly associated variables is the main aim of Factor analysis (Variation specific to a variable only muddies the water)

If we consider an factor analytic model:
$Var(X) = Var(F) + Var(e)$; or
$Var(X) = Communality + Uniqueness$
so F can be thought about as the shared (or communal) variation among our items.

Principal Axis Factoring

- What is unique to the Xs does not contribute to the analysis, so it can be thrown away and the factor analysis redone. This two step procedure is called **Principal Axis Factoring** (PFA) or **Common Factor Analysis**:
  1. Perform PCA and discard part of Var(X) not shared [i.e. discard Var(e)]
  2. Redo Factor Analysis on modified correlation matrix
- Only if communality of Xs (correlations among variables) is high will PAF be a marked improvement on PCA

Let's run a PAF on our data. Recall that most of our methods suggested a four factor model (with the Parallel analysis suggesting 3 to 4). From now on **the main focus of the output is on the factor matrix (or some ROTATED version of the factor matrix)**

Principal Axis Factoring: R code

```
# PAF with no rotation of factor matrix
EFA.none<-factanal(factanal.df, factors=4, rotations="no

#Print out factor matrix excluding |loadings| < 0.4
print(EFA.none, cutoff=0.4)
```

Remember:

- We are trying to find factors which individually describe the **four constructs:**(Physical, social, emotional and functional well being)
- Since we are excluding loadings where $-0.4 < L < 0.4$, we should (hopefully) see only one set of items correlated with any one factor

Principal Axis Factoring (no rotation): output

Loadings:

	Factor1	Factor2	Factor3	Factor4
PWB1				
PWB2		0.525		
PWB3		0.714		
PWB4		0.523		
PWB5		0.726		
PWB6		0.627		
SWB1			0.763	
SWB2			0.657	
SWB3			0.944	
SWB4				
EWB1				0.668
EWB2				0.688
EWB3				0.418
EWB4		0.416		0.409
FWB1	0.468			
FWB2	0.595			
FWB3	0.735			
FWB4	0.559			
FWB5	0.715			
FWB6	0.741			

Interpretation of results

Based on the Factor loading matrix....

- This unrotated solution is surprisingly effective
- **Factor 1** seems to be associated with **Functional WB**,
  Factor 2 with **Physical WB**,
  Factor 3 with **Social WB**, and
  Factor 4 with **Emotional WB**
- Of note, the PWB1 and SWB4 items have been excluded (i.e. association < 0.4) and the there is a **cross loading**: EWB4 loads on both factors 2 and 4, and FWB1 on factors 1 and 2

Other output included:

```
Uniquenesses:
 PWB1  PWB2  PWB3  PWB4  PWB5  PWB6
0.694 0.640 0.443 0.659 0.403 0.573
 SWB1  SWB2  SWB3  SWB4  EWB1  EWB2  EWB3
0.356 0.496 0.070 0.725 0.422 0.448 0.784
 EWB4  FWB1  FWB2  FWB3  FWB4  FWB5  FWB6
0.595 0.620 0.595 0.309 0.617 0.341 0.295
```

69.4% of item PWB1 is unique to that item (i.e. only 30.6% explained by factor model), etc.....

Rotating the factors

- One way of helping the 'components' (what we see in output) is to rotate them to align with our constructs (what we believe)
- Two main approaches:
  1. Orthogonal: Maintain linear independence between factors (Keep components perpendicular). In this case assume Physical, social, emotional and functional WB are uncorrelated
  2. Oblique: Rotate allowing factors to be correlated.

Which 'model' do you believe is more realistic??

A little background on rotation methods

Orthogonal (uncorrelated factors) rotations:

- **Varimax** ‡: Maximizes squared loadings of the items on a particular factor. Tends to lead to large or small loadings for the individual items (highly correlated variables should occur on a particular factor)
- **Quartimax**: Attempts to minimize number of factors needed to explain a particular item
- **Equimax**: A compromise between Varimax and Quartimax

Oblique (correlated factors) rotations:

- **Oblimin**: Attempts to maximize amount explained by each component (but not constrained to orthogonality)
- **Promax** ‡: Method that shrinks small factor loadings to align constructs with sets of items

Conceptualization of rotation

Let's compare orthogonal and oblique rotations using an example with a simple two factor model:

Figure: Orthogonal roation

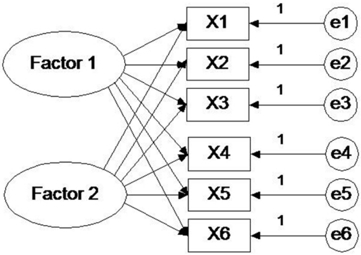
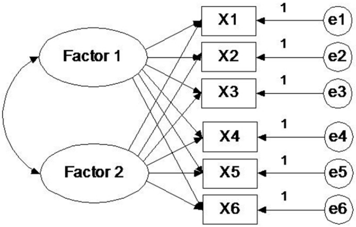


Figure: Oblique roation



Principal Axis Factoring with Varimax rotation: R code

```
#PAF with orthogonal varimax rotation
EFA.orth<-factanal(factanal.df,factors=4, rotations="var
print(EFA.orth, cutoff=0.4)
```

Exactly the same (still a PAF with 4 factors) except we change the rotation (from *"none"*) to **"varimax"**

Principal Axis Factoring with Varimax rotation: output

Loadings:

	Factor1	Factor2	Factor3	Factor4
PWB1				
PWB2		0.515		
PWB3		0.719		
PWB4		0.531		
PWB5		0.722		
PWB6		0.635		
SWB1			0.754	
SWB2			0.648	
SWB3			0.936	
SWB4				
EWB1				0.686
EWB2				0.703
EWB3				0.429
EWB4		0.410		0.435
FWB1	0.450	0.403		
FWB2	0.591			
FWB3	0.730			
FWB4	0.550			
FWB5	0.707			
FWB6	0.730			

Oblique rotation

- We can see the Varimax rotation is very similar to the unrotated solution
- In fact, it is slightly worse (FWB1 is now cross-loaded)
- Contextually, we would expect these four constructs to be correlated
- Now we will try a **Promax** rotation
- Should note that two 'factor matrices' are produced by oblique rotation. The **Pattern** matrix (which does not account for the correlations among factors), and the **Structure** matrix which does
- There is some debate about which should be interpreted. I would interpret the pattern matrix (see later) as it is much easier (tidier)

Principal Axis Factoring with Promax rotation: R code

```
#PAF with Oblique(promax) rotation
EFA.obl<-factanal(factanal.df, factors=4, rotation="prom
print(EFA.obl, cutoff=0.4)
```

Principal Axis Factoring with Promax rotation: Pattern matrix

Loadings:

	Factor1	Factor2	Factor3	Factor4
PWB1				
PWB2		0.538		
PWB3		0.795		
PWB4		0.554		
PWB5		0.781		
PWB6		0.713		
SWB1			0.772	
SWB2			0.640	
SWB3			1.013	
SWB4				
EWB1				0.733
EWB2				0.760
EWB3				0.451
EWB4				
FWB1	0.469			
FWB2	0.705			
FWB3	0.820			
FWB4	0.630			
FWB5	0.788			
FWB6	0.819			

Principal Axis Factoring with Promax rotation: Interpretation and factor correlations

From the previous slide we now have a nice solution. Our four factors coincide (cleanly) with our four constructs (Physical, Social, Emotional and Functional well being).

Some more interesting output from the PAF(Promax) is the Factor correlation matrix:

```
Factor Correlations:
        Factor1 Factor2 Factor3 Factor4
Factor1   1.000   0.134   0.582  -0.293
Factor2   0.134   1.000   0.500  -0.529
Factor3   0.582   0.500   1.000  -0.497
Factor4  -0.293  -0.529  -0.497   1.000
```

Clearly at least some of the factors are correlated:

- Factor 1 ("Functional WB") is correlated with Factor 3 ("Emotional WB")
- Factor 2 ("Physical WB") is correlated with Factors 3 and 4 ("Social WB" and "Emotional WB", respectively)
- and Factor 3 ("Social WB") with Factor 4 ("Emotional WB")

Exploratory Factor Analysis of the FACT-G data

- We have now conducted a successful **Exploratory Factor Analysis**
- Principal Axis Factoring with an oblique (Promax) rotation revealed four factors which closely coincide with the items from four different domains
- We can now name these four factors:
  1. Functional WB - 6 items (FWB1-FWB6)
  2. Physical WB - 5 items (PWB2-PWB6)
  3. Social WB - 3 items (SWB1-SWB3)
  4. Emotional WB - 3 items (EWB1-EWB3)
- We should note that 3 items (of the original 20) have been trimmed due to low association (PWB1, SWB4 and EWB4)

Now we need to go onto confirm (construct validate) this structure using a **Confirmatory Factor Analysis**

Last word on Exploratory Factor Analysis

- Since EFA is not really a hypothesis testing (inferential) analysis, we are pretty free to make choices regarding approach (extraction and/or method)
- Generally, the choices that result in the most meaningful structure of our factors, is usually the best choice
- In terms of extraction method we can use PCA, PAF and other methods (that mainly vary in how they estimate loadings)
- We can also choose various rotation methods that can help making the factors more interpretable, both orthogonal rotations (Varimax, Quartimax and Equimax) and oblique rotations (Oblimin and Promax)

No, really the last word this time

- Even though EFA (and exploratory analysis in general) does give us a lot of scope to 'massage' our analysis to get a 'useful result', there are still assumptions.
- Since all of the techniques I've discussed act on the correlation or covariance matrix (and thereafter perform eigenanalysis), there is still an assumption about relationships among variables being linear.

THANK-YOU

Questions?????